Warmstarts and other improvements in SCIP-SDP

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joint work with Marc E. Pfetsch and Stefan Ulbrich

SFB 805

Control of Uncertainty in Load-Carrying Structures in Mechanical Engineering

Mixed-Integer Semidefinite Programming

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- Mixed-integer semidefinite program

MISDP

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i \succeq 0, \\ & y_i \in \mathbb{Z} \quad \forall\, i \in \mathcal{I} \end{aligned}$$

for symmetric matrices A_i, C

- Linear constraints, bounds, multiple blocks possible within SDP-constraint

Applications

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- Combinatorial optimization problems strengthened by semidefinite relaxations
  - Max-cut / minimum-k-partitioning
  - Quadratic assignment problems (including TSP as special case)

Applications

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- Combinatorial optimization problems strengthened by semidefinite relaxations
  - Max-cut / minimum-k-partitioning
  - Quadratic assignment problems (including TSP as special case)
- Nonlinear / semidefinite problems with binary decisions
  - Robust truss topology design
  - Cardinality-constrained least squares
  - Transmission switching problems for AC power flow
  - Compressed sensing

Solving Approaches

- Cutting plane / outer approximation approaches
  - Solve LPs/MILPs and enforce SDP-constraint via cuts.
  - Most successful approach for mixed-integer second-order cone.
  - Outer approximation for SOCPs possible with polynomial number of cuts. (Ben-Tal/Nemirovski 2001)
  - Outer approximation for SDPs needs exponential number of cuts. (Braun et al. 2015)

Solving Approaches

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- Cutting plane / outer approximation approaches
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  - Outer approximation for SOCPs possible with polynomial number of cuts. (Ben-Tal/Nemirovski 2001)
  - Outer approximation for SDPs needs exponential number of cuts. (Braun et al. 2015)
- Nonlinear branch-and-bound
  - Solve SDP relaxation in each branch-and-bound node.
  - Spectral bundle or low-rank methods can be used for specific applications like max-cut, in general interior-point methods.
  - Harder to warmstart
  - Need to handle numerical difficulties in SDP-solvers

Discrete Optimization

Contents

Contents

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SCIP-SDP

- Supports both nonlinear B&B and LP-based branch-and-cut.
- Two file-readers
  - CBF
  - SDPA with added integrality information
- Constraint handler for SDP-constraints
- Relaxator for SDPs using SDPI similar to LPI
- Interfaces to three SDP solvers
  - DSDP
  - SDPA
  - MOSEK
- Two additional heuristics
  - SDP-based diving, SDP-based randomized rounding
- Two additional propagators
  - SDP-based OBBT, SDP-based dual fixing
- Parallelized version available as UG-MISDP (beta version, not yet fully stable).

Constraint Handler

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- Handles SDP-constraints in dual form

$$C - \sum_{i=1}^{m} A_i y_i \succeq 0.$$

- For branch & cut separate eigenvector cuts

$$v^\top \left(C - \sum_{i=1}^{m} A_i y_i \right) v \geq 0,$$

where v is an eigenvector to the smallest eigenvalue of $C - \sum_{i=1}^{m} A_i y_i^*$.

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  where v is an eigenvector to the smallest eigenvalue of $C - \sum_{i=1}^{m} A_i y_i^*$.
- Adds linear constraints implied by SDP-constraint during presolving (e.g., non-negativity of diagonal entries).
  - Redundant for nonlinear branch-and-bound, but can be used by SCIP during presolving for fixing variables.
  - Still lead to speedup of 6% even for nonlinear branch-and-bound.

Relaxator and SDPI

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- Relaxator solves trivial relaxations (e.g., all variables fixed), otherwise calls SDPI.
- Upper level SDPI does some local presolving important for SDP-solvers, e.g.,
  - removing fixed variables,
  - removing zero rows/columns.
- Lower level SDPI brings SDP into the form needed by the solver (e.g., primal instead of dual SDP for MOSEK) and solves it.
- In case SDP-solver failed to converge (e.g., because of failure of constraint qualification), upper level SDPI can apply penalty formulation and call lower level SDPI for adjusted problem.

Contents

Warmstarts

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- MIP: Large savings by starting dual simplex from optimal basis of parent node.
- Interior-point solvers: Need initial points $X \succ 0$ and $Z \succ 0$ for primal-dual pair

Dual SDP (D)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i = Z \succeq 0 \\ & y \in \mathbb{R}^m \end{aligned}$$

Primal SDP (P)

$$\begin{aligned} \inf \quad & C \bullet X \\ \text{s.t.} \quad & A_i \bullet X = b_i \qquad \forall\, i \leq m \\ & X \succeq 0, \end{aligned}$$

where $A \bullet B = \mathrm{Tr}(AB) = \sum_{ij} A_{ij} B_{ij}$.

Discrete Optimization

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where $A \bullet B = \mathrm{Tr}(AB) = \sum_{ij} A_{ij} B_{ij}$.

- Not satisfied by optimal solution of parent node, which will be on boundary.
- Infeasible interior-point methods do not require equality constraints to be satisfied, but still need strict positive definiteness.

$\Rightarrow$ Cannot easily warmstart with unadjusted solution of parent node.

Warmstarting Techniques

- Four different approaches implemented for SCIP-SDP 3.1.0:
  - Starting from earlier iterates
  - Convex combination with strictly feasible solution
  - Projection onto set of positive definite matrices
  - Rounding problems

Starting from Earlier Iterates

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- Proposed by Gondzio for MIP.
- Store earlier iterate further away from optimum but still sufficiently interior.
- First solve relaxation to sufficiently large gap ε_1 (e.g., 10^{-2}), then save current iterate and continue solving until original tolerance ε_2 (e.g., 10^{-5}) is reached.

Convex Combination with Strictly Feasible Solution

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- First proposed by Helmberg and Rendl, recently revisited by Skajaa, Andersen and Ye for MIP.
- Take convex combination between optimal solution (X^*, y^*, Z^*) and strictly feasible (X^0, y^0, Z^0).
- Choose (X^0, y^0, Z^0) as default initial point like $(I, 0, I)$, possibly scaled either by maximum entry of primal/dual matrix or maximum of both.
- Also possible to compute analytic center of feasible region once in root node and use this as strictly feasible solution.

Projection onto Set of Positive Definite Matrices

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- Project optimal solution of parent node onto set of positive definite matrices with $\lambda_{\text{min}} \geq \underline{\lambda} > 0$.

- For given optimal solution X^* (equivalently Z^*) of parent node let $V\text{Diag}(\lambda)V^\top = X^*$ be an eigenvector decomposition. Then compute

$$V\text{Diag}((\max\{\lambda_i, \underline{\lambda}\})_{i \leq n})V^\top.$$

Rounding Problems

- Proposed by Çay, Pólik and Terlaky for MISOCP based on Jordan Frames.
- Fix EV decomposition $V\text{Diag}(\lambda^*)V^\top = X^*$ and optimize over eigenvalues.

Rounding Problems

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- Proposed by Çay, Pólik and Terlaky for MISOCP based on Jordan Frames.
- Fix EV decomposition $V\text{Diag}(\lambda^*)V^\top = X^*$ and optimize over eigenvalues.
- First solve the **linear** primal rounding problem

Primal SDP (P)

$$\begin{aligned} \inf \quad & C \bullet X \\ \text{s.t.} \quad & A_i \bullet X = b_i \quad \forall\, i \leq m \\ & X \succeq 0 \end{aligned}$$

Primal Rounding Problem (P-R)

$$\begin{aligned} \inf \quad & C \bullet \left(V\text{Diag}(\lambda)V^\top\right) \\ \text{s.t.} \quad & A_i \bullet \left(V\text{Diag}(\lambda)V^\top\right) = b_i \quad \forall\, i \leq m \\ & \lambda_i \geq 0 \quad \forall\, i \leq n. \end{aligned}$$

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Primal Rounding Problem (P-R)

$$\begin{aligned} \inf \quad & C \bullet \left(V\mathrm{Diag}(\lambda)V^\top\right) \\ \text{s.t.} \quad & A_i \bullet \left(V\mathrm{Diag}(\lambda)V^\top\right) = b_i \quad \forall\, i \le m \\ & \lambda_i \ge 0 \quad \forall\, i \le n. \end{aligned}$$

- (P-R) is restriction of (P) to matrices with same eigenvectors as X^*

$$\Rightarrow \text{optval(P-R)} \ge \text{optval(P)} \ge \text{optval(D)}.$$

- (P-R) unbounded $\Rightarrow$ (D) infeasible
- optval(P-R) $\le$ cutoff bound $\Rightarrow$ subtree can be pruned for suboptimality

Rounding Problems

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- If (D) is not cut off, let $W\text{Diag}(\mu^*)W^\top = Z^*$ be an eigenvector decomposition of the parent node and solve the corresponding linear dual rounding problem

Dual SDP (D)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & C - \sum_{i=1}^{m} A_i y_i = Z \\ & Z \succeq 0, \quad y \in \mathbb{R}^m \end{aligned}$$

Dual Rounding Problem (D-R)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & W\text{Diag}(\mu)W^\top + \sum_{i=1}^{m} A_i y_i = C \\ & \mu_i \geq 0 \quad \forall\, i \leq n, \quad y \in \mathbb{R}^m. \end{aligned}$$

Rounding Problems

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- If (D) is not cut off, let $W\text{Diag}(\mu^*)W^\top = Z^*$ be an eigenvector decomposition of the parent node and solve the corresponding linear dual rounding problem

Dual SDP (D)

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Dual Rounding Problem (D-R)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & W\text{Diag}(\mu)W^\top + \sum_{i=1}^m A_i y_i = C \\ & \mu_i \geq 0 \quad \forall\, i \leq n, \quad y \in \mathbb{R}^m. \end{aligned}$$

- Since (D-R) is restriction of (D) to matrices with same eigenvectors as Z^*,

$$\text{optval(D-R)} \leq \text{optval(D)} \leq \text{optval(P)} \leq \text{optval(P-R)}.$$

- optval(D-R) = optval(P-R) $\Rightarrow$ problem solved to optimality

Rounding Problems

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Dual Rounding Problem (D-R)

$$\begin{aligned} \sup \quad & b^T y \\ \text{s.t.} \quad & W\text{Diag}(\mu)W^\top + \sum_{i=1}^m A_i y_i = C \\ & \mu_i \geq 0 \quad \forall\, i \leq n, \quad y \in \mathbb{R}^m. \end{aligned}$$

- Since (D-R) is restriction of (D) to matrices with same eigenvectors as Z^*,

$$\text{optval(D-R)} \leq \text{optval(D)} \leq \text{optval(P)} \leq \text{optval(P-R)}.$$

- optval(D-R) = optval(P-R) $\quad \Rightarrow \quad$ problem solved to optimality
- Otherwise use convex combination to compute strictly feasible initial point.

Implementation

- After successfully solving SDP, relaxator saves optimal solution in local „stickingatnode“ constraint.
- Auxiliary constraint only used to store data locally in node, no check/enfo etc.
- Only store X and y, recompute Z to save memory.
- Rounding problems solved via LPI.
- Different techniques enabled via parameter in relaxator.

Rounding Problems

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testset	time	roundtime	statistics for feasible roundingproblems					infeasibility	
			opt	cutoff	warmstart	pfail	dfail	detected	undetected
CLS	229.38	101.19	0.03	0.68	0.03	0.00	1847.37	310.27	841.17
MkP	271.18	6.97	0.00	0.40	0.88	0.12	188.18	1.49	459.83
TT	102.73	17.80	0.02	44.65	284.81	0.00	13,616.42	24.21	1805.33
CS	166.69	86.72	0.17	6022.54	4794.20	0.00	0.02	0.01	0.37

Run on cluster of 40 Intel Xeon E5-1620 3.5 GHz processors with 4 cores and 32GB memory; time limit of 3600 seconds; times as shifted geometric means, SDPs solved using SDPA 7.4.0; $\gamma = 0.5$.

Discrete Optimization

Comparison of Warmstarting Techniques

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settings	solved	time	sdpiter
no warmstart	290	117.85	22,827.93
unadjusted warmstart	126	821.82	–
earlier iterate: gap 0.01	172	396.93	–
earlier iterate: gap 0.5	252	213.88	26,923.91
convcomb: 0.01 scaled (pdsame) id	288	113.60	19,697.25
convcomb: 0.5 scaled (pddiff) id	289	108.60	18,307.29
convcomb: 0.5 scaled (pdsame) id	290	109.92	19,684.70
convcomb: 0.5 analcent	288	140.21	25,351.48
projection	289	112.87	20,195.03
roundingprob 0.5 id	281	180.95	16,955.37
roundingprob inf only	289	159.66	18,521.50

Run on cluster of 40 Intel Xeon E5-1620 3.5 GHz processors with 4 cores and 32GB memory; time limit of 3600 seconds; times (and iterations) as shifted geometric means (over instances solved by all settings except unadjusted warmstart and preoptimal), SDPs solved using SDPA 7.4.0.

Comparison of Warmstarting Techniques

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Speedup for conv 0.01 pdsame

testset	solved	time	sdpiter
CLS	0	-11.4 %	-19.3 %
MkP	+1	-17.2 %	-21.3 %
TT	-3	+17.5 %	+34.0 %
CS	0	-9.4 %	-18.3 %

Speedup for conv 0.5 pddiff

testset	solved	time	sdpiter
CLS	0	-6.7 %	-12.2 %
MkP	+1	-0.1 %	-10.2 %
TT	-2	+33.5 %	+2.8 %
CS	0	-27.2 %	-30.5 %

Speedup for conv 0.5 pdsame

testset	solved	time	sdpiter
CLS	-1	-9.9 %	-19.7 %
MkP	+2	-8.6 %	+0.5 %
TT	-1	+15.4 %	-5.3 %
CS	0	-13.3 %	-13.8 %

Speedup for projection

testset	solved	time	sdpiter
CLS	-1	-1.7 %	-6.4 %
MkP	+1	+5.7 %	+12.2 %
TT	-1	+7.9 %	-2.7 %
CS	0	-15.8 %	-22.1 %

Run on cluster of 40 Intel Xeon E5-1620 3.5 GHz processors with 4 cores and 32GB memory; time limit of 3600 seconds; times (and iterations) as shifted geometric means (over instances solved by all settings except unadjusted warmstart and preoptimal), SDPs solved using SDPA 7.4.0.

Contents

SDP-Knapsacks

SDP-Knapsack

$$\sup \; b^T y$$
$$\text{s.t.} \quad C - \sum_{i=1}^{m} A_i y_i \succeq 0,$$
$$y_i \in \{0,1\} \quad \forall\, i \le m$$

with $C \succeq 0$ and $A_i \succeq 0$ for all $i \le m$.

- Structure appears, e.g., in truss topology design and cardinality-constrained least squares.
- Same monotonicity structure as classical knapsack:
  - If $y \in \{0,1\}^m$ infeasible, then all $\tilde{y} \ge y$ infeasible as well.

Complexity

- Strongly $\mathcal{NP}$-hard in general
  - Multidimensional knapsack as special case for diagonal matrices

Complexity

- Strongly $\mathcal{NP}$-hard in general
  - Multidimensional knapsack as special case for diagonal matrices
- Pseudopolynomial-time algorithm for fixed matrix-dimension
  - $\mathcal{O}(mn^3(B+1)^n(2B+1)^{n(n-1)/2})$, where $B := \max_{i \leq n} C_{ii}$
  - Dynamic Programming over all possible slack matrices $Z := C - \sum_{i=1}^{m} A_i y_i$

Cover Constraints

- Knapsack Cover $\mathcal{C} \subseteq \{0,1\}^m$: If $C - \sum_{i \in \mathcal{C}} A_i y_i \not\succeq 0$.
- Minimal Knapsack Cover: If additionally $C - \sum_{i \in \mathcal{C} \setminus j} A_i y_i \succeq 0$ for all $j \in \mathcal{C}$.

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- Each Cover $\mathcal{C}$ induces a valid inequality

$$\sum_{i \in \mathcal{C}} y_i \leq |\mathcal{C}| - 1.$$

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- Each Cover $\mathcal{C}$ induces a valid inequality

$$\sum_{i \in \mathcal{C}} y_i \leq |\mathcal{C}| - 1.$$

- If $\mathcal{C}$ is minimal, this is a facet of conv $\left\{y \in \{0,1\}^{\mathcal{C}} : C - \sum_{i \in \mathcal{C}} A_i y_i \succeq 0\right\}$ (As long as $C - A_j \succeq 0$ for all $j \leq m$).

Discrete Optimization

Finding Knapsack-Covers

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- Complement of positive-semidefinite cone is non-convex.
- In particular, finding minimal cover is not equivalent to knapsack problem.

$\Rightarrow$ Greedy-Heuristic for minimal covers: Add elements until no longer positive semidefinite, then remove elements which are no longer necessary.

Finding Knapsack-Covers

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- Complement of positive-semidefinite cone is non-convex.
- In particular, finding minimal cover is not equivalent to knapsack problem.

⇒ Greedy-Heuristic for minimal covers: Add elements until no longer positive semidefinite, then remove elements which are no longer necessary.

- Alternative: Use characterization $X \not\succeq 0 \Leftrightarrow \exists w \in \mathbb{R}^m : w^\top X w < 0$ and solve for fixed w the linear knapsack problem

Cover generation

$$\begin{aligned} \min \quad & \mathbb{1}_m^\top y \\ \text{s.t.} \quad & w^\top C w - \sum_{i \leq m} \left(w^\top A_i w\right) y_i < 0 \\ & y \in \{0,1\}^m. \end{aligned}$$

Discrete Optimization

Lifting

- $\mathcal{C}$ minimal cover
- $\sigma : \overline{\mathcal{C}} \to \{1, \dots, m - |\mathcal{C}|\}$ ordering of $\overline{\mathcal{C}} := \{1, \dots, m\} \setminus \mathcal{C}$
- $$a_j := |\mathcal{C}| - 1 - \max_{y \in \{0,1\}^m} \left\{ \sum_{i \in \mathcal{C}} y_i + \sum_{i \in \mathcal{D}} a_i y_i : C - \sum_{i \in \mathcal{C} \cup \mathcal{D}} A_i y_i - A_j \succeq 0 \right\},$$

  where $\mathcal{D} := \{i \in \overline{\mathcal{C}} : \sigma(i) < \sigma(j)\}$.

  Then the lifted cover

  $$\sum_{i \in \mathcal{C}} y_i + \sum_{i \in \overline{\mathcal{C}}} a_i y_i \leq |\mathcal{C}| - 1$$

  defines a facet of conv $\left\{ y \in \{0,1\}^m : C - \sum_{i \leq m} A_i y_i \succeq 0 \right\}$.

Lifting

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  defines a facet of conv $\left\{ y \in \{0, 1\}^m : C - \sum_{i \leq m} A_i y_i \succeq 0 \right\}$.
- Need to solve MISDP (or expensive dynamic programming) to find a_j.

$\Rightarrow$ Use heuristics, like solving SDP and rounding objective down.

Contents

Conclusion & Outlook

Conclusion

- General framework for solving MISDPs
- Warmstarting is possible and can help for some applications.
- Knapsacks generalizable to SDP-case

Conclusion & Outlook

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Conclusion

- General framework for solving MISDPs
- Warmstarting is possible and can help for some applications.
- Knapsacks generalizable to SDP-case

Outlook

- Better heuristics to find and lift SDP-knapsack covers
- Implementation of SDP-knapsack constraints

Discrete Optimization

SCIP-SDP is available in source code at

`http://www.opt.tu-darmstadt.de/scipsdp/`

Thank you for your attention!

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