MINLP and Stronger Relaxation of Bilinear Terms

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SCIP Optimization Suite · http://scip.zib.de

SCIP Workshop

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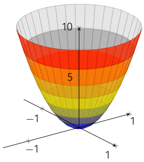
Outline

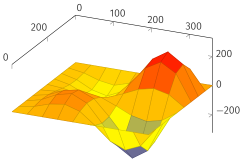
Mixed-Integer Nonlinear Programs (MINLPs)

$$\begin{aligned} \min\ & c^\mathsf{T} x \\ \text{s.t. } & g_k(x) \leq 0 && \forall k \in [m] \\ & x_i \in \mathbb{Z} && \forall i \in \mathcal{I} \subseteq [n] \\ & x_i \in [\ell_i, u_i] && \forall i \in [n] \end{aligned}$$

The functions $g_k \in C^1([\ell, u], \mathbb{R})$ can be



convex or



nonconvex

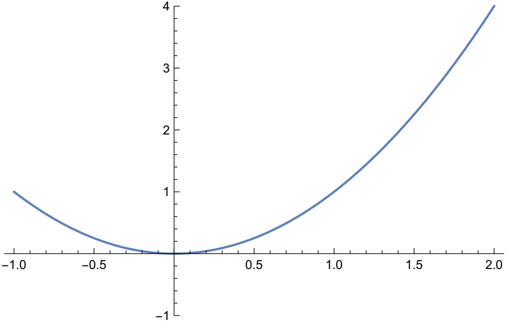
To solve to global optimality, SCIP uses LP-based spatial branch and bound

LP based spatial Branch & Bound

- Try to build a polyhedral relaxation $\mathcal{R}$
- Solve $\mathcal{R}$ and get solution x^*
- If x^* is feasible we are done. If not,
- Strengthen $\mathcal{R}$ by separating x^*
- When not possible, branch spatially (i.e., on continuous variables)

Example

Consider a constraint $y = x^2, x \in [-1, 2]$



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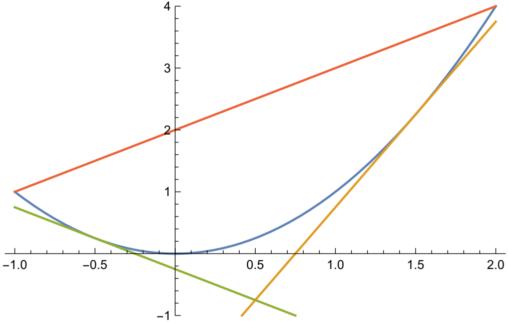

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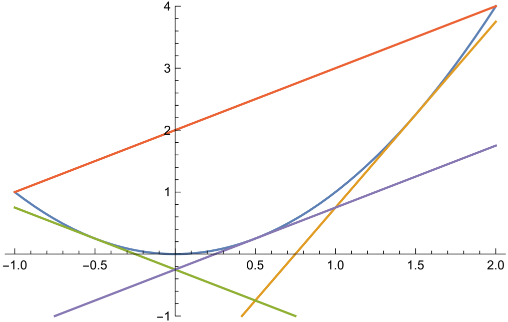

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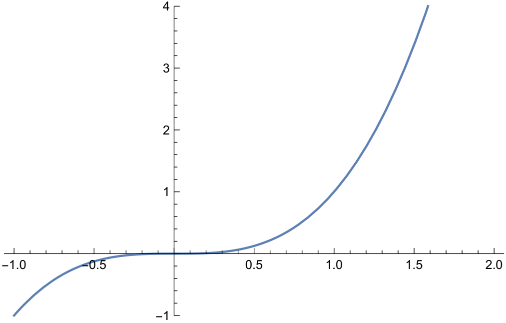

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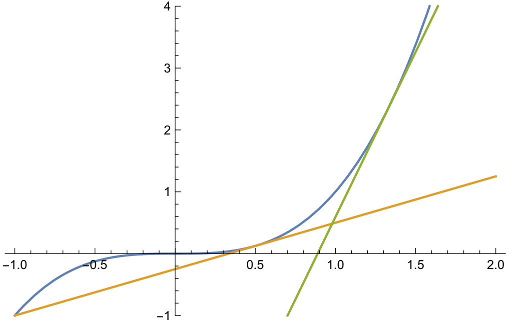

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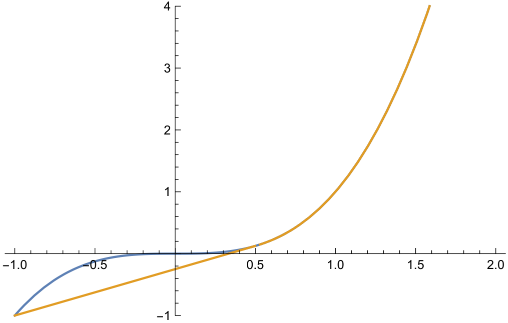
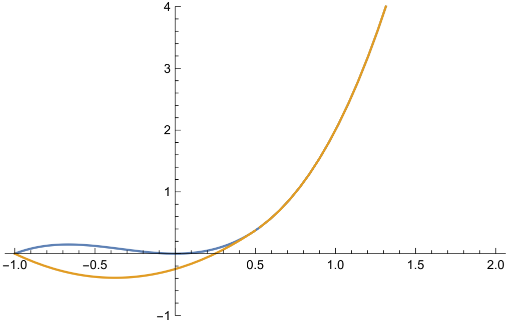


Example

- we can handle simple functions
- we can build more complicated functions by adding or multiplying

Example: Addition

- consider the sum, $y = x^2 + x^3, x \in [-1, 2]$.
- we can decompose the constraint as $w = x^2, z = x^3, y = w + z$



Example: Product

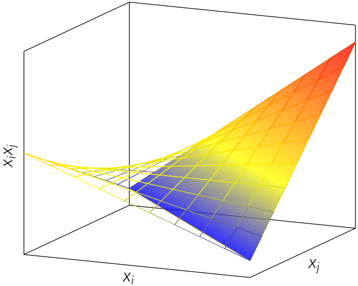
- consider the product $z = x^2y^3$.
- we can, again, decompose the constraint as $z = uv, u = x^2, v = y^3$
- what to do with $z = uv$?

McCormick Inequalities

Theorem [McCormick '76, Al-Khayyal and Falk '83]

Let $x_i \in [\ell_i, u_i]$, $x_j \in [\ell_j, u_j]$, then

$$conv(\{(X_{ij}, x_i, x_j) \mid X_{ij} = x_i x_j\}) = \left\{ (X_{ij}, x_i, x_j) \;\middle|\; \begin{array}{ll} \ell_j x_i + \ell_i x_j - \ell_i \ell_j & \leq X_{ij} \\ u_j x_i + u_i x_j - u_i u_j & \leq X_{ij} \\ \ell_j x_i + u_i x_j - u_i \ell_j & \geq X_{ij} \\ u_j x_i + \ell_i x_j - \ell_i u_j & \geq X_{ij} \end{array} \right\}.$$



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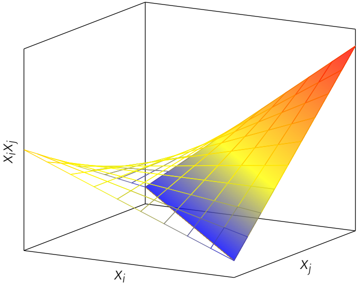

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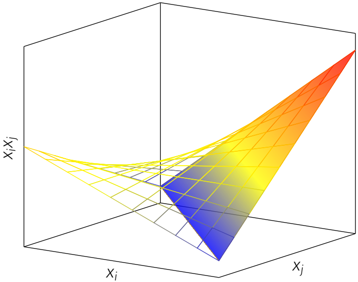

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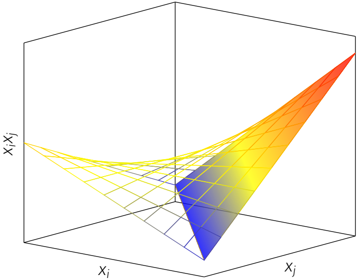

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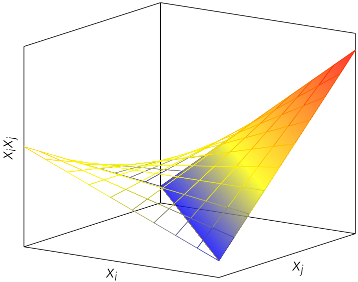

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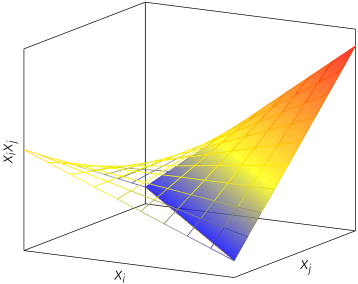

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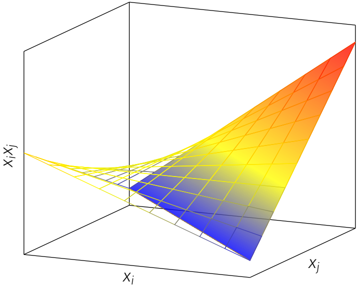

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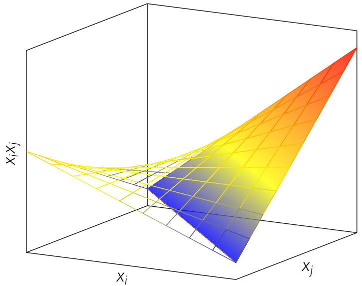

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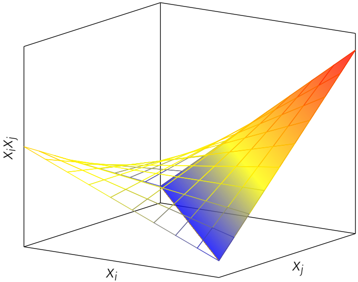

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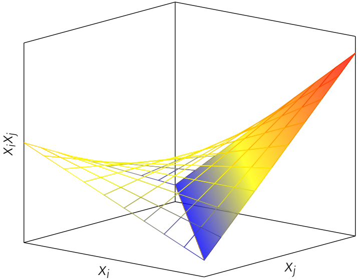

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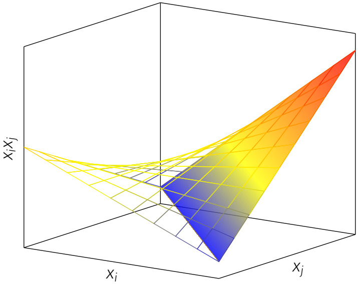

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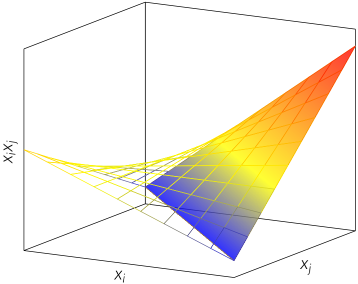

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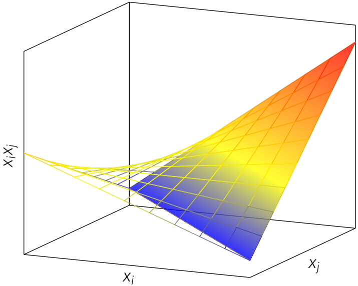

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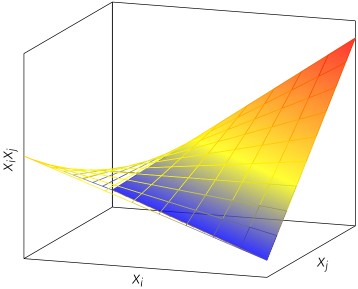


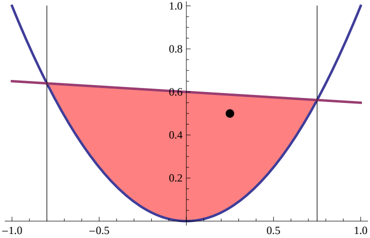
Comments

- procedure can be generalized
- polyhedral relaxation depends on the bounds of variables
- bounds are very important

Spatial Branch and bound

The variable bounds determine the convex relaxation, e.g.,

$$x^2 \le \ell^2 + \frac{u^2 - \ell^2}{u - \ell}(x - \ell) \quad \forall x \in [\ell, u].$$



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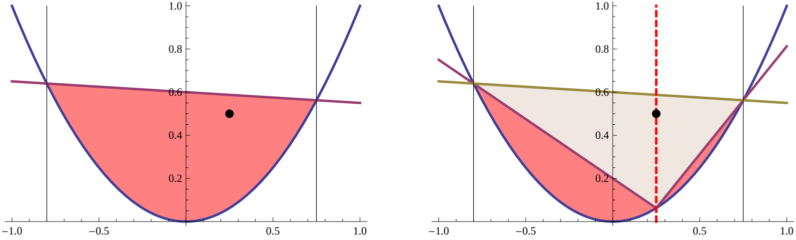
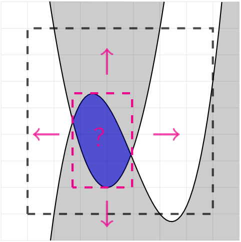
Thus, branching on a nonlinear variable in a nonconvex term allows for tighter relaxations:



Bound Tightening: OBBT

- min/max x_k $s.t.$ $x \in \mathcal{R}, c^{\mathsf{T}}x \leq z^*$
- simple and effective



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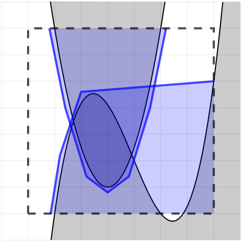

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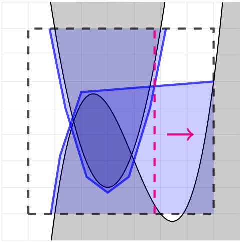

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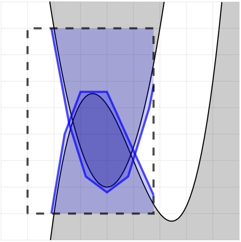
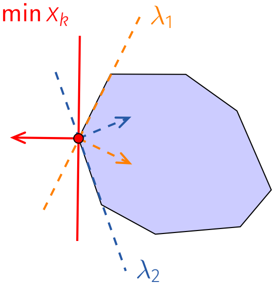


Bound Tightening: OBBT

- min/max x_k $s.t.$ $x \in \mathcal{R}, c^\top x \le z^*$
- simple and effective
- careful, might be expensive



Advanced implementation in SCIP

- fast propagation of duality certificates $x_k \ge \sum_i r_i x_i + \mu z^* + \lambda^\top b$
- greedy ordering for faster LP warmstarts
- filtering of provably tight bounds
- 16% faster (24% on instances $\ge$ 100 seconds) and less time outs [Gleixner, Berthold, Mueller, Weltge, 2017]

Outline

We can do better

- we know how to build a polyhedral relaxation
- quality depends on the bounds and we have a procedure for tightening bounds
- but, how good is the McCormick relaxation?

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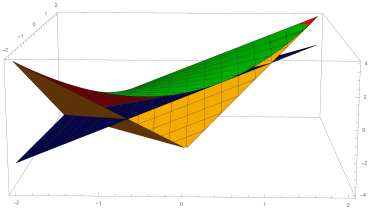
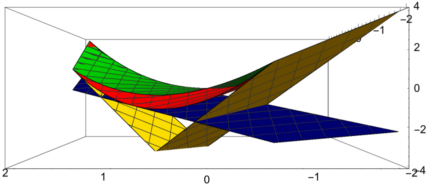
- quality depends on the bounds and we have a procedure for tightening bounds
- but, how good is the McCormick relaxation?
- Linderoth '04 suggests to branch along the diagonals of $[\ell_i, u_i] \times [\ell_j, u_j]$





green — graph of $X_{ij} = x_i x_j$
yellow — McCormick relaxation
red — $conv_P(x_i x_j)$: convex envelope over $P \rightsquigarrow$ significantly tighter

Idea

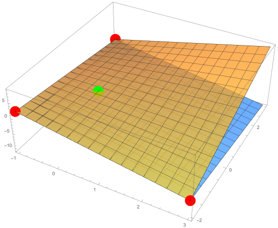
Try to find a valid $P \subsetneq [\ell_i, u_i] \times [\ell_j, u_j]$ and study
$conv\{(x_i, x_j, X_{ij}) \mid X_{ij} = x_i x_j, (x_i, x_j) \in P\}$

Two ingredients:

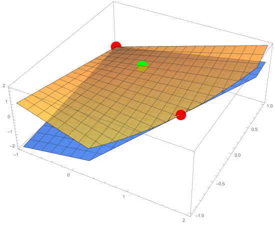
- given P, convex envelope of $X_{ij} = x_i x_j$ over P?
- how to find P?

Convex envelope: Locatelli ’16

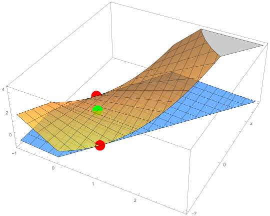
- Three cases when computing tangent inequality of $conv_P(x_i x_j)$ at reference point:



- select three vertices of P



- select a vertex and a point on a facet of P



- select two points on two facets of P

Easy to compute tangent inequalities of $conv_P(x_i x_j)$ if vertices and facets of P are explicitly given.

Before computing P...

- Some problems readily give us P, e.g., `pointpack08` from MINLPLib2
- 56 bilinear terms
- 8 linear constraints $x_i \leq x_j$
- How does exploiting P perform?

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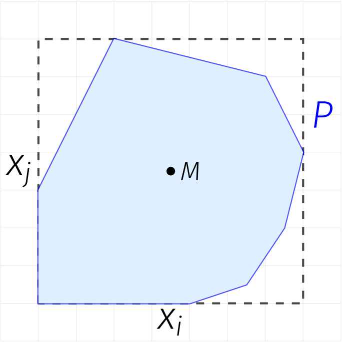
	SCIP default		SCIP$^+$	
perm.	nodes	time	nodes	time
1	1844k	1279	71k	64
2	1780k	1186	69k	80
3	1486k	943	87k	72
4	744k	483	193k	168
5	1372k	1054	89k	93
6	370k	277	45k	57
7	2285k	1522	60k	66
8	168k	136	53k	56
9	1771k	1174	45k	55
10	190k	145	48k	50

- on average 10x less time and 15x less nodes

Computing P

- $P = proj_{(x_i,x_j)}(\mathcal{R})$ is "best" possible, but impractical
- after OBBT, $M := (\frac{u_i+\ell_i}{2}, \frac{u_j+\ell_j}{2}) \in P$
- facets intersecting segment joining M with each corner



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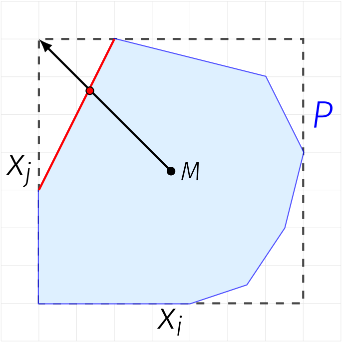
Optimize along directions from M to each corner



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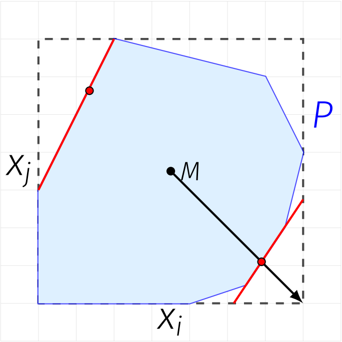

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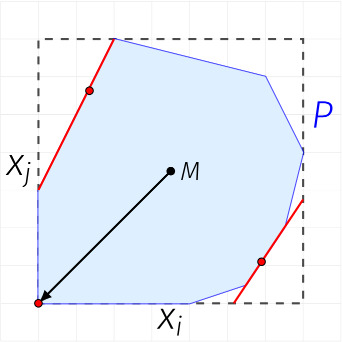

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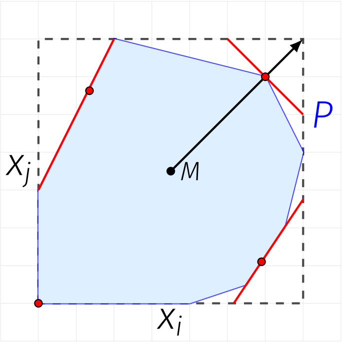

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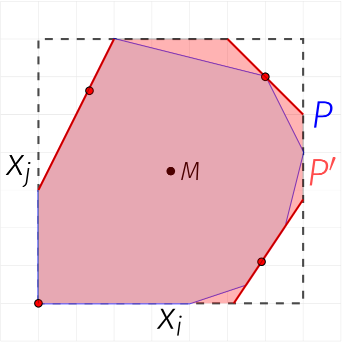

After 4 LPs

$P' \supseteq P = proj_{(x_i,x_j)}(\mathcal{R})$ described by at most

- 4 nontrivial inequalities
- 4 axis-parallel inequalities

Details

E.g. from $M = (M_i, M_j) := (\frac{\ell_i+u_i}{2}, \frac{\ell_j+u_j}{2})$ to (u_i, u_j) via the LP

$$\max\left\{\alpha : \begin{pmatrix} x_i \\ x_j \end{pmatrix} = \begin{pmatrix} M_i \\ M_j \end{pmatrix} + \alpha \begin{pmatrix} u_i - M_i \\ u_j - M_j \end{pmatrix}, x \in \mathcal{R}, \alpha \in [0,1]\right\}$$

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$$\max\left\{x_i : \frac{x_i - M_i}{u_i - M_i} = \frac{x_j - M_j}{u_j - M_j}, x \in \mathcal{R}\right\}$$

Essentially OBBT with one additional constraint

Connection to OBBT

Can use all OBBT tricks from Gleixner, Berthold, Mueller, Weltge ’17:

LP filtering

- E.g., if $\exists(x^*, X^*) \in \mathcal{R} : x_i^* = \ell_i \wedge x_j^* = u_j \Rightarrow$ filter $M \to (\ell_i, u_j)$
- already applicable during standard OBBT

LP ordering

- exploit simplex warmstart
- e.g., after $\max\{x_i \mid (x, X) \in \mathcal{R}\}$ solve all $M \to (u_i, \cdot)$ (dual feasible LP basis)

Updating facets with new incumbent information

- new incumbent solution tightens P if $\lambda \neq 0$
- similar to *Lagrangian Variable Bounds* from Gleixner and Weltge ’13

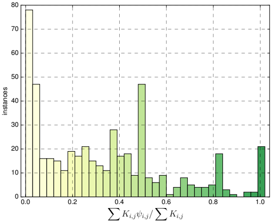
Outline

Computational results: affected instances

- 1367 instances of MINLPLib2 after presolving
- filter instances
  - with no bilinear terms
  - solved before finding facets
  - with no facets found

$\rightsquigarrow$ 422 potentially affected instances

Frequency of nontrivial facets:



- $K_{i,j} := |\{k \in [m] \mid (Q_k)_{i,j} \neq 0\}|$ occurrences of a bilinear term $x_i x_j$
- $\psi_{i,j} := \mathbb{1}(\text{found facet for } x_i x_j)$
- final measure: $\sum_{i,j} K_{i,j}\psi_{i,j} / \sum_{i,j} K_{i,j} \in [0, 1]$

Computational results: gap closed

SCIP settings

- `propagating/obbt/itlimitfactor = -1`
- provide best known solution
- `limits/restart = 0` and `limits/totalnodes = 1`
- `separation/emphasis/aggressive = TRUE`

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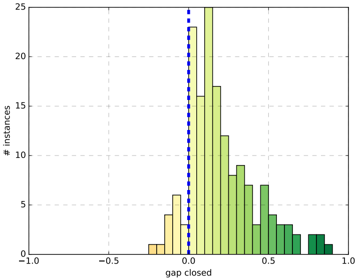
Root node results

	instances	gap closed
all	422	16%
>1% change	171	40%
>1% better	155	44%
>1% worse	16	-15%



Computational results: performance

Setup[1]

- `limits/time = 1800s`
- `limits/gap = 1e-4`
- 6 different permutations
- one instance per cluster node

		default	no propagation		no prop. + no sepa.	
	n	# solved	# solved	time	# solved	time
`ALL`	422	213	205	1.07	199	1.12
`[1,tlim]`	143	138	131	1.17	128	1.30
`[10,tlim]`	96	91	84	1.26	81	1.52
`[100,tlim]`	45	40	34	1.36	34	1.76

Results

- +14 more solved instances
- 12% speed-up on `ALL`
- 76% speed-up on `[100,tlim]`

[1] Intel(R) Xeon(R) CPU E5-2660 v3 @ 2.60GHz

Outline

Conclusion

Summary

- very brief introduction to LP-based branch and bound
- tighter convexification over projections of the feasible region

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- 40% root gap closed on affected instances
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- more drastic speedups on specific instances

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- tighter convexification over projections of the feasible region

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Thank you very much for your attention!

MINLP and Stronger Relaxation of Bilinear Terms

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SCIP Optimization Suite · http://scip.zib.de

SCIP Workshop

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